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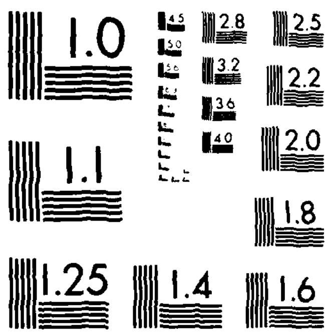

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Mathematical techniques are developed to analyze the nonlinear wave interactions that occur in both elastic and visco-plastic solids. Representations are also derived to describe small amplitude stress waves in materials whose transmitting properties vary in space and time.

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3. Seymour, B. R. and Varley, E, Backlund transformations for the nonlinear telegraph equation. Publication pending.

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8. SCIENTIFIC PERSONNEL SUPPORTED BY THIS PROJECT AND DEGREES AWARDED DURING THIS REPORTING PERIOD:

PROFESSOR ERIC VARLEY - PRINCIPAL INVESTIGATOR

PROFESSOR BRIAN SEYMOUR - CONSULTANT

1. FORWARD

The dynamic responses of materials whose physical properties vary in space and time is extremely complex and difficult to analyze, especially when these materials are subjected to large dynamic loads. Even when the material properties are piecewise uniform, as in laminates, the problem of how to arrange the component materials so that the broad qualitatative features of the dynamic response of the material can be controlled is not yet fully resolved.

As a start on this problem, E.Varley and his co-workers have developed mathematical techniques,(see [1]-[4]), that can be used to analyse the diverse nonlinear wave interactions that can occur when a slab of elastic-plastic material is finitely deformed by plane waves propagating in directions normal to the parallel interfaces bounding the slab.The slab could be contained between two other different elastic-plastic materials that are of semi-infinite extent in the direction of wave propagation,or the slab could be just one layer in a multi-layered structure.One important feature of the proceedures that were developed is that, in the absence of strong entropy gradient,the deformation at the boundaries which seperate the different layers can be determined independantly of the deformation at interior points of the layers.This result holds even in the presence of stong shock waves.

The techniques developed by E.Varley and his co-workers depended on the fact that the dynamic responses of many materials could be approximated by a family of stress-strain laws for which the governing nonlinear equations could be solved analytically. This yields representations that can be used to study complicated wave interactions. The materials include polycrystalline solids at pressures up to the yield stress,metals when subjected to pressures in the hydrodynamic range,water,explosive products,gases,as well as elastic-plastic,rigid-plastic,and rigid-elastic materials.

The general aim of the proposed program of research was to continue the development of mathematical techniques that can be used to analyze the interactions of large amplitude waves in diverse materials. Special attention was to be given to those interactions that result when strong waves propagate through visco-plastic materials, as well as through materials whose properties vary in space and time.

2. SUMMARY OF COMPLETED RESEARCH

2.1 Waves in visco-plastic materials.

The dynamic responses of many materials are strongly rate dependent, even in deformations that are produced by impact. Although the combined effects of nonlinearity and viscosity are well understood at the head of a wave, (see, for example, Varley and Rogers [5] and Varley and Seymour [6]), there are few theoretical investigations which describe conditions far inside the wave. As a start on improving this situation we have analyzed wave motions in visco-plastic materials for which the equations relating the stress $\sigma(X,t)$, the strain $\lambda(X,t)$, and the material velocity $u(X,t)$ satisfy the equations

$$\frac{\partial\sigma}{\partial X} = \rho_0\frac{\partial u}{\partial t}, \quad \frac{\partial u}{\partial X} = \frac{\partial\lambda}{\partial t} \qquad (1)$$

and

$$\frac{\partial\lambda}{\partial t} = \frac{1}{E(\sigma)}\frac{\partial\sigma}{\partial t} + \omega(\sigma). \qquad (2)$$

We have shown that for certain special forms of the material functions $E(\sigma)$ and $\omega(\sigma)$, which correspond to physically resonable behavior, the system of nonlinear equations (1) and (2) can be transformed into a system of linear equations. More precisely, it has been shown that there exist functions $Y(\sigma,u)$ and $S(\sigma,u)$ such that if

$$\bar{X} = X - Y(\sigma,u) \text{ and } \bar{t} = t - S(\sigma,u) \qquad (3)$$

, rather than X and t, are used as independent variables then $\sigma(\bar{X},\bar{t})$ and $u(\bar{X},\bar{t})$ satisfy the linear equations

$$\frac{\partial\sigma}{\partial\bar{X}} = \rho_0\frac{\partial u}{\partial\bar{t}} \text{ and } E_0\frac{\partial u}{\partial\bar{X}} = \frac{\partial\sigma}{\partial\bar{t}} + \frac{\sigma}{\tau_0} \qquad (4)$$

where E_0 and τ_0 are constants. This fact has been used in Seymour and Varley [7] and in Varley and Seymour [8] to investigate the combined effects of nonlinearity and viscosity in a wide variety of deformations, such as that produced during sudden impact.

2.2 Elastic waves in materials whose properties vary in space and time.

We have also investigated the propagation of stress pulses in materials whose transmitting properties vary with X and t. This problem arises, for example, when the pulses are produced during the rapid heating of a material for which the transmitting properties are strongly dependent on the temperature.It also occurs in the study of the small amplitude vibrations of a bar which is also transmitting large amplitude stretching waves.

The simplest situation to consider is when the disturbance is governed by the linear wave equation

$$c^2(X,t)\frac{\partial^2 w}{\partial X^2} = \frac{\partial^2 w}{\partial t^2} \qquad (5)$$

where the variation of the sound speed c(X,t) is known in terms of the fluctuations in the properties of the transmitting materials.

There are no known general techniques that can be used to construct solutions to the linear partial differential equation (5).In the special case when c depends only on one of the independent variables, sometimes transform techniques can be used to replace the partial differential equation for w by an ordinary differential equation for its transform.Even then, because the ordinary differential equation contains variable coefficients, usually it is impossible to find a representation for the transform that can be inverted to obtain w(X,t).To make any headway with the general problem approximate proceedures must be used

The best known approximate proceedure works when c(X,t) is slowly varying in the deformations which are being studied.These proceedures are those used in the theory of geometric acoustics in which w(X,t) is expanded in an asymptotic series in some small parameter, ε, which measures how slowly c(X,t) is varying.What is not commonly realised is that for certain forms of c(X,t) these series either terminate, or can be summed ,to obtain representations for w(X,t) that are valid for <u>all</u> ε. Accordingly,for these special forms of c(X,t) it is possible to construct the general solution to equation (7).

In Seymour and Varley [9] we show that the geometric acoustic expansion terminates after the <u>first</u> term whenever c(X,t) satisfies an equation of the form

$$\frac{\partial^2}{\partial X^2}[M(c)] = \frac{\partial^2}{\partial t^2}[N(c)] \qquad (6)$$

where M and N are certain functions.The general solution to the nonlinear equation (6) was obtained together with the corresponding general solutions to equation (5).It was shown that there are forms of c(X,t) satisfying (6) which model both the quantitative and the qualitatative behaviour of many real systems that transmit acostical disturbances.Also,the corresponding representations that are obtained for $w(X,t)$ can be used to analyze some rather complicated wave motions in materials whose physical properties are varying in space and time.Another paper on this subject is in preparation.

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